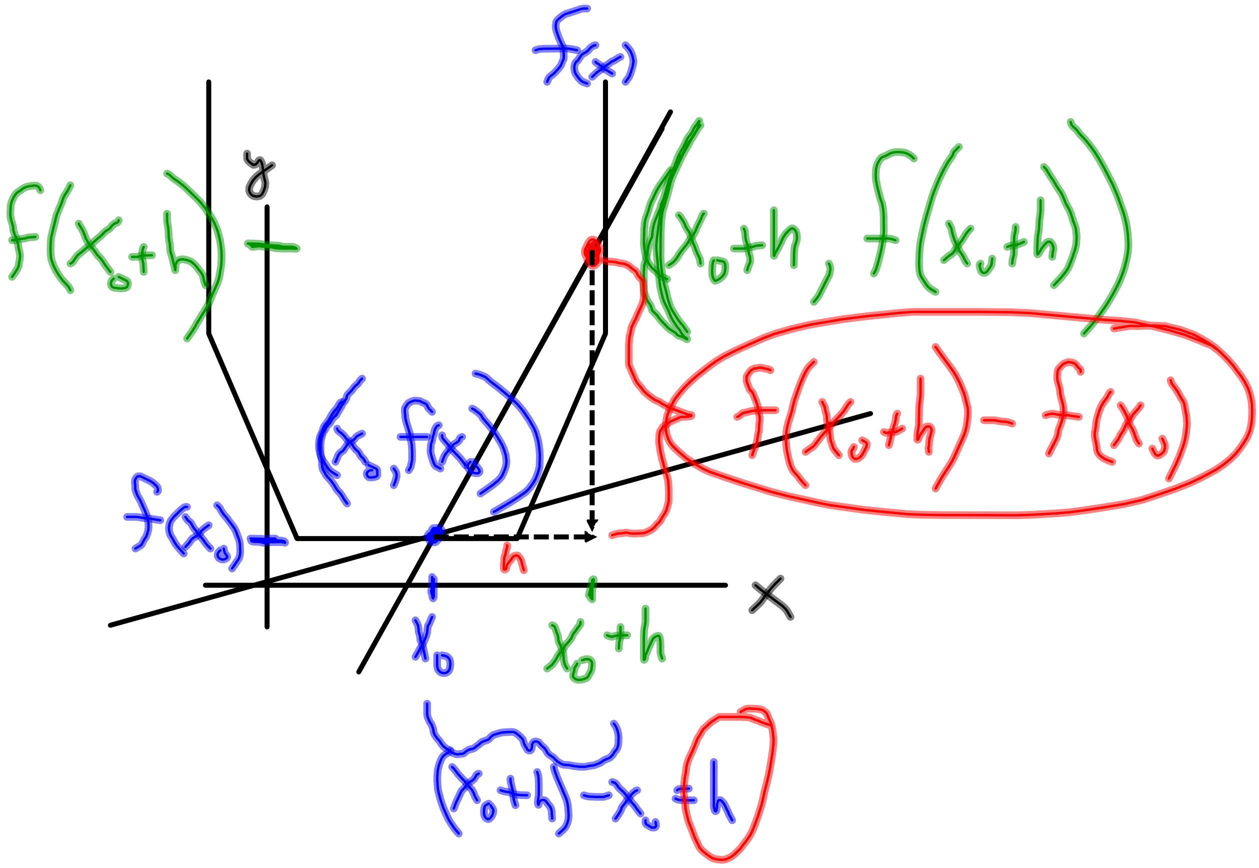
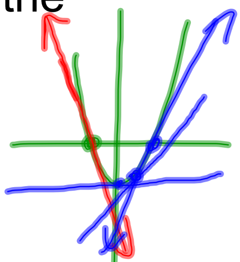
3.1 Differentiation



$$m_{tan} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

Find an equation for the slope of the tangent line and then evaluate at the given point.

$f(x) = x^2 - 1$; $x_0 = -1$



$m = 2x_0$

$0 = 2x_0$

$$m_{tan} = \lim_{h\to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$(x_0+h)(x_0+h)$

$$= \lim_{h\to 0} \frac{[(x_0+h)^2 - 1] - [(x_0)^2 - 1]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x_0^2} + 2hx_0 + h^2 \cancel{-1} - \cancel{x_0^2} \cancel{+1}}{h}$$

$$= \lim_{h\to 0} \frac{2hx_0 + h^2}{h}$$

$$= \lim_{h\to 0} 2x_0 + h$$

$(-1, 0)$

$$= 2x_0$$

$y = mx + b$

$0 = -2(-1) + b$

$-2 = b$

$m = -2$

$b = -2$

$$\boxed{y = -2x - 2}$$

ex/ find an equation for the tangent line to the parabola $y=2x^2-4$ at the point $(1,-2)$

$$m_{tan}=\lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$$

0

$$=\lim_{h\to 0}\frac{[2(x_0+h)^2-4]-[2(x_0)^2-4]}{h}$$

$$=\lim_{h\to 0}\frac{2(x_0^2+2hx_0+h^2)-4-2x_0^2+4}{h}$$

$$=\lim_{h\to 0}\frac{2x_0^2+4hx_0+2h^2-2x_0^2}{h}$$

$$=\lim_{h\to 0}\frac{4hx_0+2h^2}{h}$$

$$=\lim_{h\to 0}4x_0+2h$$

$m(x_0)=4x_0$

$m(1)=4(1)=4$

$y=mx+b$

$m=4$

$b=-6$

$\boxed{y=4x-6}$

$y=mx+b$

$(-2)=(4)(1)+b$

$-6=b$

The derivative of a function f at a point x_0,
denoted $f'(x_0)$, is

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

provided this limit exists.